Quiz 3A, Calculus I - No Calculators

Dr. Graham-Squire, Spring 2014

8:34
8:38
4

Name: Key

1. (4 points) Use implicit differentiation to find y'.

$$y^2\left(\frac{\sec x}{y^2} = x^3 + 2y^4\right)y^2 \checkmark$$

$$\Rightarrow \frac{d}{dx}\left(\sec x = x^3y^2 + 2y^6\right)$$

$$\sec x \tan x = 3x^2y^2 + x^3(2y)y' + 12y^5 \cdot y'$$

2.5 for derivative

$$\sec x \tan x - 3x^2y^2 = (2x^3y + 12y^5)\,y' \checkmark$$

1.5 for simplifying

$$\boxed{\frac{\sec x \tan x - 3x^2y^2}{2x^3y + 12y^5} = y'} \checkmark$$

or

$$\frac{d}{dx}\left(\frac{\sec x}{y^2} = x^3 + 2y^4\right)$$

$$\frac{(\sec x)(\tan x)\cdot y^2 - 2yy'\cdot \sec x}{y^4} = 3x^2 + 8y^3\cdot y'$$

$$y^2(\sec x)(\tan x) - 2y\sec x\, y' = 3x^2y^4 + 8y^7\cdot y'$$

$$y^2(\sec x)(\tan x) - 3x^2y^4 = 8y^7y' + 2y\sec x\, y'$$

$$\boxed{\frac{(\sec x)y^2(\tan x) - 3x^2y^4}{8y^7 + 2y\sec x} = y'}$$

Don't need to simplify

0.5

2. (3 points) Find $f'(x)$ if $f(x) = \sqrt{\cot\left(\frac{\pi}{4}x\right)}$ $= \left(\cot\left(\frac{\pi}{4}x\right)\right)^{1/2}$

0.5

$$f'(x) = \boxed{\frac{1}{2}\left(\cot\left(\frac{\pi}{4}x\right)\right)^{-1/2} \cdot \left(-\csc^2\left(\frac{\pi}{4}x\right)\cdot\frac{\pi}{4}\right)}$$

✓ ✓

3. (3 points) Find the equation of the tangent line at $x = -1$ if $f(x) = \arctan(x^5)$.

$$f'(x) = \frac{1}{1+(x^5)^2} \cdot 5x^4 = \frac{5x^4}{1+x^{10}}$$ ✓ ✓

$$f'(-1) = \frac{5(-1)^4}{1+(-1)^{10}} = \frac{5}{2}$$ ✓

$$f(-1) = \arctan(-1)^5 = \arctan(-1) = -\frac{\pi}{4}$$ ✓

$$y - y_1 = m(x - x_1)$$

$$y - \left(-\frac{\pi}{4}\right) = \frac{5}{2}\left(x - (-1)\right)$$

$$\boxed{y = \frac{5}{2}x + \frac{5}{2} - \frac{\pi}{4}}$$ ✓ ✓

Quiz 3B, Calculus I - No Calculators

Dr. Graham-Squire, Spring 2014

Name: Key

1. (3 points) Find the equation of the tangent line at $x = -1$ if $f(x) = \arctan(x^3)$.

$$f'(x) = \frac{1}{1+(x^3)^2} \cdot 3x^2 = \frac{3x^2}{1+x^6}$$

$$m = f'(-1) = \frac{3(-1)^2}{1+(-1)^6} = \frac{3}{2}$$

$$y_1 = f(-1) = \tan^{-1}(-1)^3 = \tan^{-1}(-1) = \frac{-\pi}{4}$$

Tangent line is:

$$\boxed{y - \left(-\frac{\pi}{4}\right) = \frac{3}{2}\left(x - (-1)\right)}$$

or $\quad y = \frac{3}{2}(x+1) - \frac{\pi}{4}$

2. (4 points) Use implicit differentiation to find y'.

$$\frac{\csc x}{y^3} = x^2 + 2y^5$$

$$\Rightarrow \frac{d}{dx}\left(\csc x = y^3x^2 + 2y^8\right)$$

$$-\csc x \cot x = 3y^2y'x^2 + y^3(2x) + 16y^7 \cdot y'$$

$$-\csc x \cot x - 2xy^3 = 3y^2y'x^2 + 16y^7y'$$

$$-\csc x \cot x - 2xy^3 = (3x^2y^2 + 16y^7)y'$$

$$\Rightarrow \boxed{y' = \frac{-\csc x \cot x - 2xy^3}{3x^2y^2 + 16y^7}}$$

or $\frac{d}{dx}\left(\frac{\csc x}{y^3} = x^2 + 2y^5\right) \Rightarrow \frac{-\csc x \cot x\, y^3 - 3y^2y'\csc x}{y^6} = 2x + 10y^4y'$

$$-\csc x \cot x\, y^3 - 3y^2y'\csc x = 2xy^6 + 10y^{10}y'$$

$$-\csc x \cot x\, y^3 - 2xy^6 = (3y^2\csc x + 10y^{10})y'$$

$$\boxed{\frac{-\csc x \cot x\, y^3 - 2xy^6}{3y^2\csc x + 10y^{10}} = y'}$$

3. (3 points) Find $f'(x)$ if $f(x) = \sqrt{\tan\left(\frac{\pi}{2}x\right)}$. You don't have to simplify your answer.

$$f(x) = \left(\tan\left(\frac{\pi}{2}x\right)\right)^{1/2}$$

$$f'(x) = \boxed{\frac{1}{2}\left(\tan\left(\frac{\pi}{2}x\right)\right)^{-1/2} \cdot \sec^2\left(\frac{\pi}{2}x\right) \cdot \frac{\pi}{2}}$$